Journal of Engineering Physics and Thermophysics, Vol. 73, No. 6, 2000

HEAT TRANSFER IN PLUG FLOW OF A FIBROUS SUSPENSION IN A TUBE

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UDC 532.546

The problem of heat transfer in plug flow of a fibrous suspension in a straight round tube with a constant wall temperature is solved.

Three regimes of motion of a fibrous suspension in a tube are distinguished: plug, mixed, and turbulent flows [1, 2]. In plug flow, the entire mass of fibers moves in the form of a plug, which is surrounded by a thin layer of pure liquid phase, adjacent to the solid wall. Aqueous suspensions of natural fibers (cellulose, wood pulp) are most widely used industrially as a semifinished product of pulp and paper making. They possess a high capacity for absorbing to absorb water, so that in a water-saturated fiber water usually takes more than 70% of the volume [3]. For this reason such thermophysical parameters of aqueous suspensions of natural fibers with a low concentration (up to 4–5% by mass) as thermal conductivity and heat capacity differ little from the same parameters of water. However, a comparison of heat-transfer coefficients calculated from the corresponding formulas for plug flow of water [4] in a round tube with the actual values of the heat-transfer coefficients in similar plug flow of a fibrous suspension [5] (as can be seen from Fig. 1) demonstrates not only a quantitative but also a qualitative difference in the results.

One of the main reasons for the discrepancy of the results apparently lies in the fact that in calculating the heat-transfer coefficient in plug flow of a fibrous suspension in complete conformity with the problem formulation of [4] the plug was considered to be a homogeneous solid, whereas it is actually a highly porous network of fibers (for example, for a mass concentration $c = 5\%$ the network porosity is about 0.88) submerged in the liquid. The presence of the fibers in the suspension and the deformation of the network formed by the fibers in plug flow [1, 6, 7] should definitely affect heat-exchange processes; therefore, an attempt to consider heat transfer with allowance for the factors indicated is natural.

Suppose that, in a straight round tube with a diameter $d = 2r_0$, a fibrous suspension of low concentration (up to 4–5% by mass) moves in the regime of steady-state plug flow. In a first approximation the mean flow velocity w coincides with the plug velocity [6, 7].

We adopt the cylindrical coordinate system r, φ, x: r is the distance from the tube axis; the x axis is directed along the tube axis in the direction of the flow. Assuming that for $x \le 0$ a steady-state plug flow with a constant temperature T_0 over the cross section is formed, we consider the heat-exchange process in a semi-infinite tube $x > 0$ with a constant wall temperature $T_{\rm w}$.

In order to take into account the influence of the structure of the fibrous suspension on the heat-transfer process, we specify the thermal-conductivity coefficient λ in the fiber plug by a dependence of the form

$$\lambda = \lambda_0 \left(1 - k \left|\tau_{xr}\right|\right) , \tag{1}$$

assuming that the constants λ_0 and k depend on the suspension concentration. A dominant influence of the tangential stresses τ_{xr} introduced into formula (1) on the deformation of the network of fibers is shown in [6, 7].

The moduli of the stresses τ_{xr} satisfy the equality

$$\left|\tau_{xr}\right| = \frac{r}{2} \left|\frac{\partial p}{\partial x}\right| = \frac{r}{r_0} \tau_{\rm w} . \tag{2}$$

Petrozavodsk State University, Petrozavodsk, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 73, No. 6, pp. 1181-1186, November–December, 2000. Original article submitted June 24, 1999; revision submitted November 9, 1999.

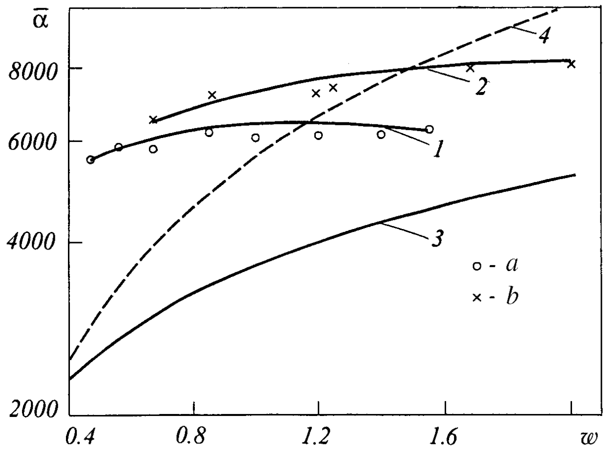

Fig. 1. Dependence of the mean heat-transfer coefficient $\bar{\alpha}$ on the mean velocity w: 1 and 2) result of calculation by formula (22) for a fibrous suspension [1) 2.1%; 2) 2.8%]; 3) water (plug flow); 4) water (actual flow) [5]; points, result of the experiment of [5] for a fibrous suspension [a) 2.1%; b) 2.8%]. $\bar{\alpha}$, W/(m^2·K); w, m/sec.

With account for equalities (1) and (2) the energy equation in plug flow of a medium with temperature-independent parameters [4] has the form

$$\frac{w}{a_0}\frac{\partial T}{\partial x} = \left(1 - k\tau_{\text{w}}\frac{r}{r_0}\right)\frac{\partial^2 T}{\partial r^2} + \left(1 - 2k\tau_{\text{w}}\frac{r}{r_0}\right)\frac{1}{r}\frac{\partial T}{\partial r}, \tag{3}$$

$$a_0 = \frac{\lambda_0}{\rho c_p}. \tag{4}$$

We introduce the dimensionless temperature Θ and the dimensionless coordinates X and R:

$$\Theta = \frac{T - T_{\text{w}}}{T_0 - T_{\text{w}}}, \quad X = \frac{4}{\text{Pe}}\frac{x}{d}, \quad R = \frac{r}{r_0}, \quad \text{Pe} = \frac{wd}{a_0}. \tag{5}$$

Equation (3) takes the form

$$\frac{\partial \Theta}{\partial X} = (1 - k\tau_{\text{w}}R)\frac{\partial^2 \Theta}{\partial R^2} + \left(\frac{1}{R} - 2k\tau_{\text{w}}\right)\frac{\partial \Theta}{\partial R}. \tag{6}$$

The boundary conditions for the function $\Theta(X, R)$ with a constant wall temperature with allowance for the problem symmetry are as follows [4]:

$$\Theta(0, R) = 1, \quad \frac{\partial \Theta(X, 0)}{\partial R} = 0, \quad \Theta(X, 1) = 0. \tag{7}$$

To solve Eq. (6), we use the method of expansion in a small parameter, which showed itself well in solving problems of determination of the resistance in plug flow [6, 7]. As this parameter β we take the ratio

$$\beta = \frac{\tau_{\text{w}}}{\mu}. \tag{8}$$

As shown in [6, 7], the dimensionless parameter β characterizes the shear deformation of the fiber network at the tube wall.

Introducing the notation $M = \mu k$ and taking account of formula (8), we can write Eq. (6) in the form

$$\frac{\partial\Theta}{\partial X} = (1 - \beta MR)\,\frac{\partial^2\Theta}{\partial R^2} + \left(\frac{1}{R} - 2\beta M\right)\frac{\partial\Theta}{\partial R}\,. \tag{9}$$

The solution of Eq. (9) is sought as a series in powers of β:

$$\Theta = \Theta^{(0)} + \Theta^{(1)}\beta + \Theta^{(2)}\beta^2 + \dots\,. \tag{10}$$

Substituting series (10) into Eq. (9) and equating terms of the same powers of β, we obtain equations for the functions $\Theta^{(0)}$ and $\Theta^{(1)}$:

$$\frac{\partial\Theta^{(0)}}{\partial X} = \frac{\partial^2\Theta^{(0)}}{\partial R^2} + \frac{1}{R}\,\frac{\partial\Theta^{(0)}}{\partial R}\,, \tag{11}$$

$$\frac{\partial\Theta^{(1)}}{\partial X} = \frac{\partial^2\Theta^{(1)}}{\partial R^2} + \frac{1}{R}\,\frac{\partial\Theta^{(1)}}{\partial R} - M\left(R\,\frac{\partial^2\Theta^{(0)}}{\partial R^2} + 2\,\frac{\partial\Theta^{(0)}}{\partial R}\right). \tag{12}$$

The boundary conditions for the function $\Theta^{(0)}(X, R)$ obviously coincide with those for the function $\Theta(X, R)$; therefore, the solution of Eq. (11) can be written immediately [4] as

$$\Theta^{(0)}(X, R) = \sum_{n=1}^{\infty} A_n J_0(\varepsilon_n R)\exp(-\varepsilon_n^2 X)\,, \quad A_n = \frac{2}{\varepsilon_n J_1(\varepsilon_n)}\,. \tag{13}$$

Substitution of solution (13) into Eq. (12) gives

$$\frac{\partial\Theta^{(1)}}{\partial X} = \frac{\partial^2\Theta^{(1)}}{\partial R^2} + \frac{1}{R}\,\frac{\partial\Theta^{(1)}}{\partial R} - g(X, R)\,,$$

$$g(X, R) = M\sum_{n=1}^{\infty} A_n\varepsilon_n\,[\varepsilon_n R J_0(\varepsilon_n R) + J_1(\varepsilon_n R)]\exp(-\varepsilon_n^2 X)\,. \tag{14}$$

The function $\Theta^{(1)}(X, R)$ has only zero boundary conditions:

$$\Theta^{(1)}(0, R) = 0\,, \quad \frac{\partial\Theta^{(1)}(X, 0)}{\partial R} = 0\,, \quad \Theta^{(1)}(X, 1) = 0\,. \tag{15}$$

To find the solution of Eq. (14) with boundary conditions (15), we use a known method [8]. Confining ourselves to solutions in the class of functions to which, together with the first derivatives with respect to X and R and the second derivatives with respect to R, the Laplace transform with respect to the coordinate X can be applied [9], we perform the following transformation on each term of Eq. (14):

$$\hat{F}(\xi, R) = \int_0^{\infty} F(X, R)\exp(-\xi X)\,dX\,.$$

Here and subsequently Laplace transforms will be denoted by a caret the functions. As a result of the transformation with account for the first boundary condition of (15), Eq. (14) takes the form

$$\frac{\partial^2 \hat{\Theta}^{(1)}}{\partial R^2} + \frac{1}{R}\frac{\partial \hat{\Theta}^{(1)}}{\partial R} - \xi \hat{\Theta}^{(1)} - \hat{g}(\xi, R) = 0 ,$$

$$\hat{g}(\xi, R) = M \sum_{n=1}^{\infty} \frac{A_n \varepsilon_n}{\xi + \varepsilon_n^2} [\varepsilon_n R J_0(\varepsilon_n R) + J_1(\varepsilon_n R)] . \tag{16}$$

It is evident that after the transformation the boundary conditions (15) remain zero.

We will seek the solution of Eq. (16) as the series

$$\hat{\Theta}^{(1)}(\xi, R) = M \sum_{k=1}^{\infty} \hat{b}_k(\xi) J_0(\varepsilon_k R) , \tag{17}$$

where $\hat{b}_k(\xi)$ are functions to be determined. The solution suggested satisfies the transformed second and third boundary conditions of (15). The first boundary condition was allowed for in the derivation of Eq. (16).

Substitution of series (17) into Eq. (16) leads to the equality

$$M \sum_{k=1}^{\infty} (\xi + \varepsilon_k^2) \hat{b}_k(\xi) J_0(\varepsilon_k R) = \hat{g}(\xi, R) .$$

Multiplying both its sides by $RJ_0(\varepsilon_i R)$, $i = 1, 2, ...$, and integrating over R from 0 to 1 with allowance for the definition of the function $\hat{g}(\xi, R)$ by the second formula of (16), we obtain

$$\hat{b}_i(\xi) = \frac{1}{(\xi + \varepsilon_i^2) G_i} \sum_{n=1}^{\infty} \frac{A_n g_{in}}{\xi + \varepsilon_n^2} ,$$

$$G_i = \int_0^1 J_0^2(\varepsilon_i R) R dR ,$$

$$g_{in} = \int_0^1 [\varepsilon_n^2 R J_0(\varepsilon_n R) + \varepsilon_n J_1(\varepsilon_n R)] J_0(\varepsilon_i R) R dR . \tag{18}$$

Having substituted expressions (18) into series (17) and then taking the inverse Laplace transform, we find

$$\Theta^{(1)}(X, R) = M \sum_{k=1}^{\infty} b_k(X) J_0(\varepsilon_k R) , \tag{19}$$

$$b_k(X) = \frac{1}{G_k} \left[A_k g_{kk} X \exp(-\varepsilon_k^2 X) + \sum_{\substack{\nu=1 \\ \nu \neq k}}^{\infty} \frac{A_\nu g_{k\nu}}{\varepsilon_\nu^2 - \varepsilon_k^2} (\exp(-\varepsilon_k^2 X) - \exp(\varepsilon_\nu^2 X)) \right] .$$

Confining ourselves to the first two terms in series (10), after substitution of solutions (13) and (19) into it and replacement of the quantities β, M, and A_k by the expressions that define them, we obtain the temperature distribution $\Theta(X, R)$ in the fiber plug:

$$\Theta (X, R) = \sum_{n=1}^{\infty} \left[\frac{2}{\varepsilon_n J_1 (\varepsilon_n)} \exp (- \varepsilon_n^2 X) + k \tau_{\mathrm{w}} b_n (X) \right] J_0 (\varepsilon_n R) . \tag{20}$$

The average temperature of the tube over the cross section $\overline{\Theta}(X)$ is determined by the equality

$$\overline{\Theta} (X) = 2 \int_0^1 \Theta (X, R) \, R dR =$$

$$= \sum_{n=1}^{\infty} \frac{2}{\varepsilon_n} \left[\frac{2}{\varepsilon_n} \exp (- \varepsilon_n^2 X) + k \tau_{\mathrm{w}} b_n (X) \, J_1 (\varepsilon_n) \right] . \tag{21}$$

Thus, the problem posed is solved. In Fig. 1 the average heat-transfer coefficient $\overline{\alpha}$ over the portion of the tube $0 \le x \le l$ calculated from the formula [4]

$$\overline{\alpha} = - \frac{\rho c_p w d}{4l} \ln \overline{\Theta} (l) , \tag{22}$$

is compared with the corresponding values found experimentally [5]. In the calculations, we used the following data [5]: the working medium was an aqueous suspension of bleached kraft pulp; the concentration of the suspensions was 2.1 and 2.8% by mass; the temperature was 30°C; the tube diameter was 25.3 mm; the length of the heat-transfer portion was 0.25 m. Moreover, the values of τ_{w} required for calculating $\overline{\Theta}$ from formula (21) are also taken as experimental data [5]. As calculational values we used the following ones: $\nu = 0.805 \cdot 10^{-6}$ m^2/sec; $c_p = 4178$ J/(kg·K); $\rho = 1003.1$ kg/m^3, $\lambda_0 = 5.1690$ W/(m·K), and $k = 0.0518$ Pa^{-1} for $c = 2.1\%$; $\rho = 1005.7$ kg/m^3, $\lambda_0 = 4.3721$ W/(m·K), and $k = 0.0220$ Pa^{-1} for $c = 2.8\%$.

For comparison, in Fig. 1 we give dependences $\overline{\alpha}(w)$ for plug (curve 3) and actual (curve 4) water flows. In the first case, the values of $\overline{\alpha}(w)$ are calculated from the same formulas (21) and (22) but for $k = 0$ and $\lambda_0 = \lambda_{\mathrm{wat}} = 0.613$ W/(m·K). Curve 4 is plotted from data of [5].

The agreement between the experimental and calculated results is quite satisfactory. It should be noted that, first, for small flow velocities the presence of the fibers in water increases sharply the heat transfer even compared to that in the actual (turbulent) water flow. And, second, in the plug flow of the fibrous suspension the heat-transfer coefficient depends but slightly on the flow velocity, whereas in the plug flow of water it grows rather noticeably with increase in the velocity.

The smaller values of λ_0 and k for $c = 2.8\%$ compared to those for $c = 2.1\%$ have a quite natural explanation. The coefficient λ_0 characterizes the heat conduction of the suspension in the absence of deformation of the fiber network ($k = 0$). As the suspension concentration grows, the velocities of the relative motion of the phases decrease, thereby decreasing the heat conduction of the suspension, which is fixed by the decrease in λ_0. The coefficient k characterizes the change in the heat conduction due to the applied tangential stresses, i.e., due actually to the deformation of the fiber network. With increase in the concentration, the deformation of the fiber network for the same stresses decreases, reducing the influence of this factor on the heat conduction, which is manifested as a decrease in the coefficient k.

NOTATION

a_0, parameter determined by formula (4); A_n, coefficients of series (13); $b_k(X)$, coefficients of series (19); $\hat{b}_k(\xi)$, coefficients of series (17); c, concentration of the suspension; c_p, isobaric heat capacity of unit mass of the suspension; d, tube diameter; $F(X, R)$ and $\hat{F}(\xi, R)$, inverse transform and Laplace transform; G_i and g_{in}, quantities determined by the second and third formulas of (18); J_0 and J_1, Bessel functions of the first kind of zero and first orders; k, coefficient in formula (1); l, length of the calculated portion of the tube; M, coefficient in Eq. (9); p, pressure; Pe, Péclet number; r, radial coordinate; r_0, tube radius; R, dimensionless radial coordinate; T, temperature at the flow point; T_0, temperature in the initial cross section of the tube; T_w, wall temperature; x, longitudinal coordinate; X, dimensionless longitudinal coordinate; $\bar{\alpha}$, average heat-transfer coefficient; β, dimensionless coefficient determined by formula (8); ε_n, roots of the function J_0; Θ, dimensionless temperature; $\Theta^{(0)}$, $\Theta^{(1)}$, ..., functions entering into expansion (10); $\bar{\Theta}$, average dimensionless temperature over the cross section; λ, thermal-conductivity coefficient of the suspension; λ_0, coefficient in formula (1); μ, coefficient entering into the equations that determine the relation between the stresses and the elastic deformations of the fiber network [6, 7]; ν, kinematic viscosity of the liquid phase of the suspension; ξ, variable of the Laplace transform; ρ, suspension density; τ_{xr}, tangential stresses at the point; τ_w, modulus of tangential stresses on the tube wall. Subscripts: i, k, n, ν, natural numbers 1, 2, ...; w, wall.

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